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Commentary: Thinking nonlinearly about aortic biomechanics

Konstadinos A. Plestis, MD, and
Keshava Rajagopal, MD, PhD

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CENTRAL MESSAGE

Mathematical modeling of the aorta requires careful attention to mathematical rigor, physical accuracy, and biological relevance.

> *Mutationem motus proportionalem esse vi motrici impressae, et fieri secundum lineam rectam qua vis illa imprimatur.*
>
> (The change in momentum is proportional to the motive force impressed, and in a direct line along which the force is impressed.)
>
> —Isaac Newton, *Philosophiae Naturalis Principia Mathematica* (1687)

> *Ut tensio, sic vis.*
>
> (As the extension, so the force.)
>
> —Robert Hooke (1678)

Perhaps the most important concept in classical/macroscopic mechanics is the idea that a net force (independent variable) causes a change in motion (dependent variable) of a body, and that the nature of this functional relationship is mediated by the material properties of the body being subjected to the force. The canonical formulation of this principle is Newton's Second Law of Motion, commonly taught in the setting of constant mass as F = ma, or perhaps more accurately conveying the causality of the relationship, a = F/m. In particle mechanics, mass is the only material characteristic of the body. This concept from particle mechanics then extends to relatively more modern continuum mechanics, which is formulated in tensors.

In continuum mechanics, due to the applied forces on the body, the body deforms, and the manner in which it deforms depends upon material properties. For elastic solid bodies, these material response relationships—termed constitutive relationships—are expressed between stress (which expresses a combination of body forces such as gravity, and contact forces through [normal] and along [shear] surfaces and strains [displacements]). Two points are relevant here with respect to nonlinearity. First, for large deformations (eg, those in the range of a few % or more), strains include nonlinear terms that depend upon displacement *gradients*. These nonlinear terms cannot be ignored in the setting of large deformations, whereas they may be in the setting of infinitesimal deformations. Second, there may be nonlinear relationships *between* stresses and strains. For homogeneous isotropic linearly elastic solids (aka, Hookean) undergoing small deformations, the relationship $\sigma = E\varepsilon$ is used, with σ being stress, ε being a "linearized" strain, and E being the Young's modulus (a specific case of the "elastic modulus" for linearized elasticity).

Material properties are the "black box" in the relationship between forces and motions, insofar as they too, like forces, are the other independent variable input causally regulating

From the Division of Cardiac Surgery, Department of Surgery, Sidney Kimmel Medical College, Thomas Jefferson University, Philadelphia, Pa.

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Address for reprints: Keshava Rajagopal, MD, PhD, Division of Cardiac Surgery, Department of Surgery, Sidney Kimmel Medical College, Thomas Jefferson University, 1025 Walnut St, Suite 607 College, Philadelphia, PA 19107 (E-mail: keshava.rajagopal@jefferson.edu).

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motion. However, as a general matter, they are the *unknown variable* in this relationship. Whereas forces and motions are *measured*, material properties are *calculated*. Importantly, as a definitional matter, material properties are *intrinsic* to the material. They may spatially vary in inhomogeneous and anisotropic materials, but this is distinct from a dependence upon *geometric* features of the body. Nonlinear materials exhibit nonlinear relationships between stress and strain, and there is no Young's modulus for such materials (so-called neo-Hookean materials have a single constant called a shear modulus that is not termed the Young's modulus). For example, σ may $= k\varepsilon_g^2$ for some material for which ε_g is a generalized as opposed to linearized strain, where k is a constant. But if one erroneously uses a linear model and applies it to this nonlinear material, what this mathematically equates to is stating that $E = k\varepsilon_g$. One would then incorrectly conclude that there exists an E that is a function of ε_g, when "E" does not truly exist in the context of an accurate model, only k does. Furthermore, and at least as important, cardiovascular tissues do not undergo small deformations. They undergo large deformations. In such cases, the strain includes nonlinear terms that depend upon the displacement *gradient*. In the cases of large deformations, these terms cannot be ignored, whereas these nonlinear terms are often ignored in "linearized" elasticity applied to small deformations (examples of materials undergoing small deformations are metals such as steel).

In this issue of *JTCVS Open*, Eliathamby and colleagues[1] have investigated purported relationships between "aortic mechanical properties" and "aortic geometry" in the context of aortic aneurysmal disease. They studied patients with aortic root and ascending thoracic aortic aneurysms who had preoperative aortic imaging and subsequently underwent operative treatment. The aortic specimens obtained at the time of operation were subjected to biaxial loading of known/prespecified magnitude and direction. The authors found that larger-diameter aneurysms exhibited greater extents of "energy loss," or dissipation of the energy of blood (this in turn being overwhelmingly derived from left ventricular systolic function), although no relationships between diameter and "elastic modulus" or "delamination strength" were identified. No relationships between aneurysm length and any of the 3 aforementioned material parameters were identified. The authors conclude that, broadly speaking, aortic geometry and aortic mechanical properties poorly correlate. But we know this from clinical experience as well, because: (1) many large aortic aneurysms do not rupture or dissect, and (2) many small aortic aneurysms do (although of course highly variable aortic loading conditions contribute to these observations as well).

However, as discussed previously, mechanical properties are definitionally independent of geometry. Why *would* one suspect that they correlate? (See to follow as to why this could be the case.) There are at least 4 inter-related issues with the current study, ranging from the conceptual to the interpretation of data.

1. It is unclear what mathematical model the authors are using to capture aortic behavior. The authors ascertain "energy loss" and "elastic modulus," but these are in fact incompatible and mutually exclusive terms. Elastic materials do not dissipate energy.
2. Discussed previously, linearized elasticity can only apply to small deformations, whereas cardiovascular tissues undergo large deformations.
3. Also discussed previously, based upon an incorrect choice of model, one could potentially incorrectly identify (as the authors did in the case of aneurysm diameter and "energy loss") a relationship between 2 variables, when one variable does not really exist (eg, a single-valued "elastic modulus" in a nonlinearly elastic material).
4. To whatever extent correlations between aortic material properties and aortic geometry could exist, they are and must be (definitionally, as previously) only that: correlations without causation. The pathophysiological mechanisms that underlie abnormal aortic mechanics likely are the same—or at least substantially overlapping—with those that underlie aortic dilatation. That is to say, a larger-diameter aortic aneurysm may have different material properties than when that same aneurysm was a smaller diameter, but that does not mean that the material properties *depend* upon the diameter. Again, material properties are definitionally independent of material geometry.

Reference

1. Eliathamby D, Keshishi M, Ouzounian M, Forbes T, Tan K, Simmons C, et al. Aortic Symposium 2022: ascending aortic geometry and its relationship to the biomechanical properties of aortic tissue. *J Thorac Cardiovasc Surg Open*. 2023;13:32-44.